Universal Aspects of Quantum-Critical Dynamics In and Out of Equilibrium

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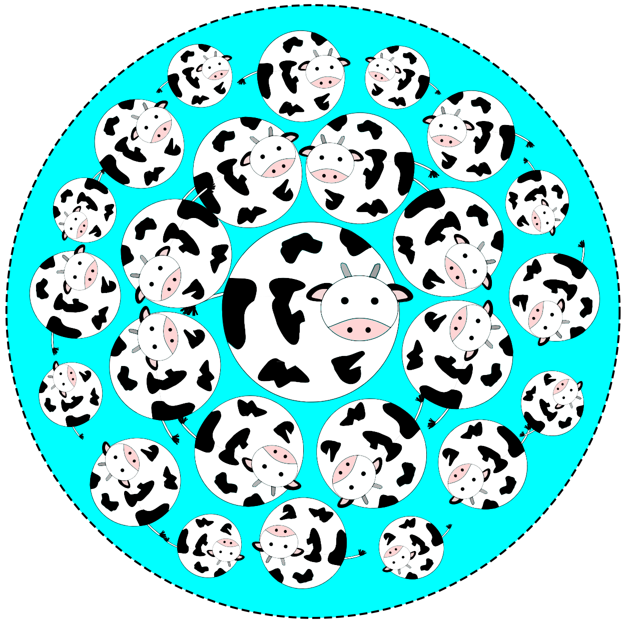
Thesis Defense

Tuesday, April 28, 2020



Outline

0. Motivation
1. NMR Relaxation in Ising Spin Chains
2. Non-equilibrium Dynamics in the Sachdev-Ye-Kitaev Model
3. Thermalization and Chaos in a Conformal Gauge Theory
4. Bounding Transport in Disordered Holographic Matter
5. Outlook and Other Work

Criticality, Dynamics, and Quantum Mechanics

- How do dynamical quantities i.e. multi-time correlation functions, transport coefficients, relaxation rates, etc. behave near critical point?
- For classical phase transitions, separate theory of dynamical phenomena
- Separation of fast and slow quantities, conservation laws put in by hand
- Quantum systems *inherently* dynamical
- Unitary time evolution encodes symmetries and conservation laws
- For systems near Quantum criticality *interplay* of thermal and quantum fluctuations control dynamics

What is a Quantum Phase Transition?

- Transition between quantum phases by varying physical parameter (magnetic field, chemical potential, etc.) at T=0

- No thermal fluctuations, driven by quantum fluctuations

- What characterizes second order QPT?

  - Diverging correlation length $\xi \sim \Lambda |g - g_c|^{\nu}$

  - Vanishing energy scale $\Delta \sim J|g - g_c|^{z\nu} \sim \xi^{-z}$

  - Dynamic critical exponent z different scaling of space and time

Example: Transverse Field Ising Model

- 1d ferromagnetic Ising chain with transverse on-site field
- $H_I = -J\sum_{i=1}^{L}[\sigma_i^z\sigma_{i+1}^z + h\sigma_i^x]$
- $h=0$ two degenerate ground states: $|\uparrow\rangle = \prod_i |\uparrow\rangle_i$ or $|\downarrow\rangle = \prod_i |\downarrow\rangle_i$
- $h \gg J$ to $O(1/h)$ one ground state: $|0\rangle = \prod_i |\rightarrow\rangle_i$
- Intermediate h, not diagonal in either σ_i^z or σ_i^x basis QM tunneling induces spin flips even at $T=0$!

T=0 Phase Diagram

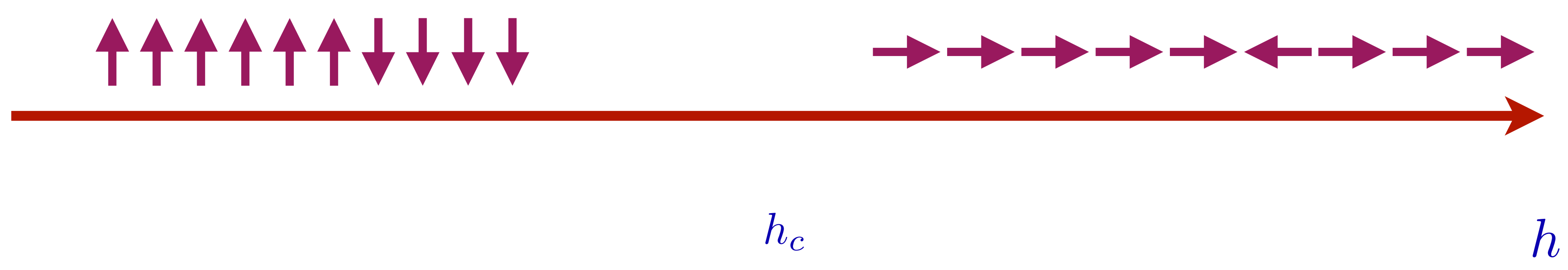
- $\mathbb{Z}_2$ symmetry spontaneously broken by $h = 0$ ground state

- Must be QPT between two phases

  - $h < h_c$: Quantum ordered/ferromagnet with $\langle 0 | \sigma_i^z | 0 \rangle = \pm N_0$

  - $h > h_c$: Quantum disordered/paramagnet with $\langle 0 | \sigma_i^z | 0 \rangle = 0$

- Transition occurs at $h_c = 1$, consequence of self-duality



Quantum Classical Mapping and Imaginary Time

- $T = 0$ TFIC phase diagram looks similar to classical Ising model but for $d = 2$

- Quantum/Classical mapping: "equivalence" $Z \sim Tr\left[e^{-\frac{H_Q}{k_B T}}\right]$ for classical system in d+1 dim with quantum system in d dim+*imaginary* time

- Interested in quantities related to correlation functions (in equilibrium) $C(t_1, t_2) = \langle A(t_1)B(t_2)\rangle = \dfrac{1}{Z} Tr\left[e^{-\beta H} e^{iHt_1} A e^{-iHt_1} e^{iHt_2} B e^{-iHt_2}\right]$

- Wick rotation: $it \to \tau$, where $\tau \in (0, \beta)$

- Calculate quantities in imaginary time and analytically continue to real time at the end

Finite temperature Signatures of Quantum Criticality

- Many systems with QCPs have no phase transition for $T > 0$, ξ always finite

- New timescale τ_φ from quantum fluctuations related to a lifetime of single particle (quasiparticle) excitations

- For $T \to 0$, $\tau_\varphi \to \infty$ scaling very important for dynamics

- Away from QCP $\tau_\varphi \gg \mathscr{C}\dfrac{\hbar}{k_B T}$ (parametrically larger), semiclassical dynamics

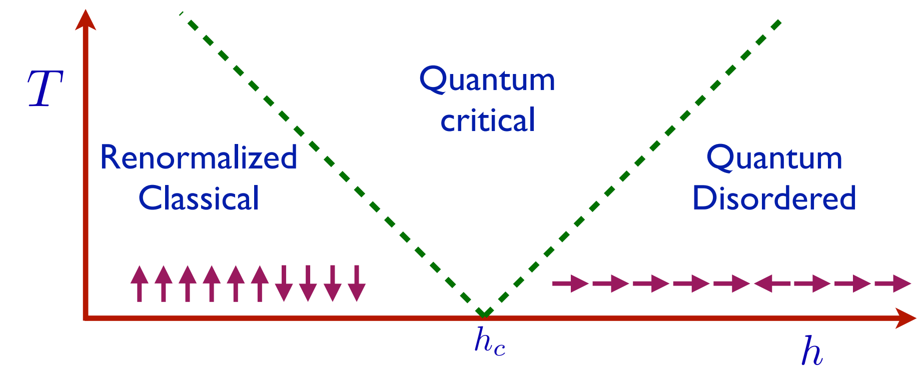
- Near QCP $\tau_\varphi \geq \mathscr{C}\dfrac{\hbar}{k_B T}$ thermal and quantum effects happen on same timescale

Example: Transverse Field Ising Model

- Dynamics captured in correlator

$$C(x_\ell, t) = \langle \sigma^z(x_\ell, t)\sigma^z(0,0)\rangle = \frac{1}{Z}Tr\left[e^{-\beta H_I}e^{iH_I t}\sigma^z_\ell e^{-iH_I t}\sigma^z_0\right]$$

- Three regimes separated by crossovers
    - RC: dilute domain wall excitations
    - QD: dilute spin flip excitations
    - QC: no single particle excitations
- Universal scaling form $C(x,t) = ZT^{1/4}\Phi_I\left(\frac{Tx}{c}, Tt, \frac{\Delta}{T}\right)$



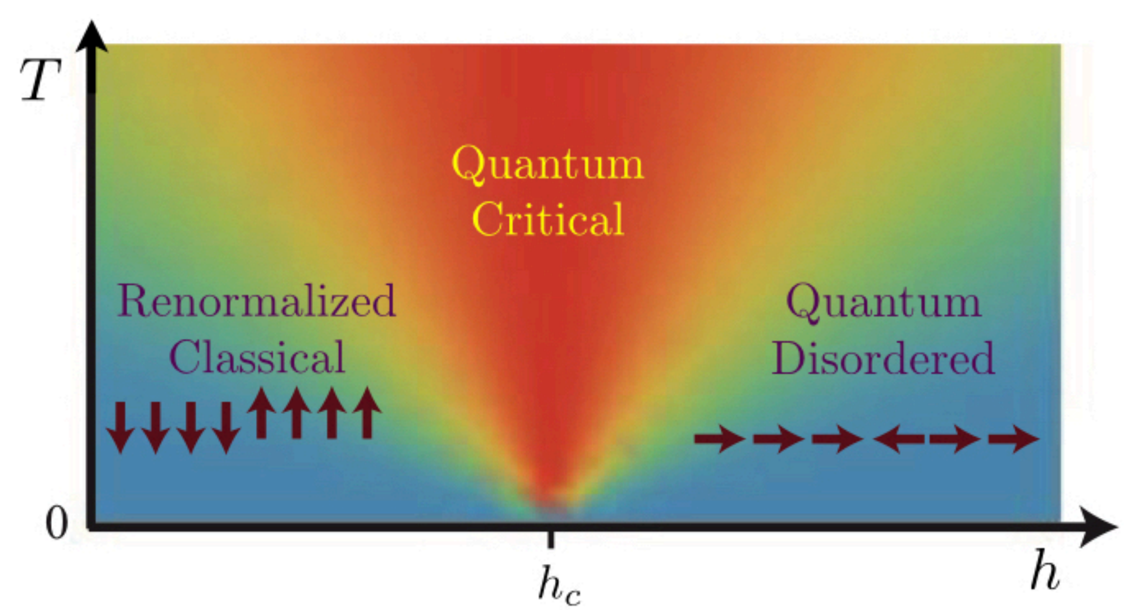
	RC	QD	QC
Regime	$T \ll \Delta$	$T \ll -\Delta$	$T \gg \Delta$
ξ	$\sim c/\lvert\Delta\rvert$	$\sim \frac{ce^{\Delta/T}}{\sqrt{\lvert\Delta\rvert T}}$	$\sim \frac{c}{T}$
τ_φ	$\sim \frac{e^{\Delta/T}}{T}$	$\sim \frac{e^{\lvert\Delta\rvert/T}}{T}$	$\sim \frac{1}{T}$

How can we study Quantum Critical Dynamics in *Real* Time?

- In general, perturbative expansions and analytic continuation do not commute
- Q/C mapping only captures correlations during time period $\hbar/k_B T$, results for $\omega \ll T$ generally untrustworthy
- What are some ways to study dynamics in interacting QC systems?
  - Map interacting system to free theory (1)
  - Large N expansion (2, 3)
  - Keldysh real time formalism (2)
  - AdS/CMT Correspondence, Hydrodynamics (4)

NMR Relaxation in Ising Spin Chains

Based on J.S., N.P. Armitage, F. H.L. Essler, and S. Sachdev."*NMR relaxation in Ising spin chains*", Phys. Rev. B. **99**, 035156 (2019), arXiv:1810.11466.



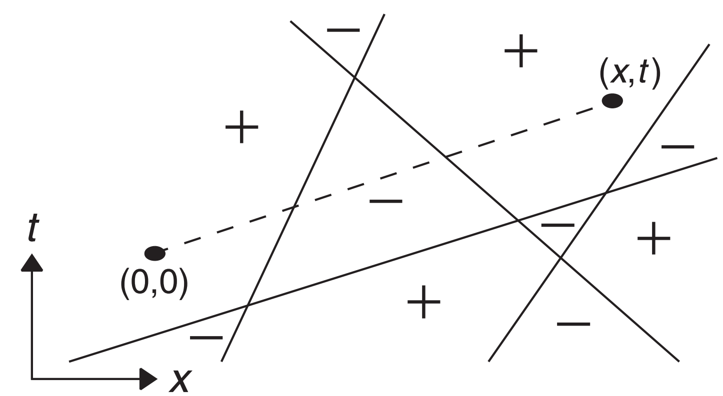
Finding Fermions

- The TFIC, an interacting spin system can be mapped onto system of free fermions!
- Spins represented as $\sigma^x_\ell = 1 - 2c^\dagger_\ell c_\ell,\ \sigma^z_\ell = -\prod_{\ell'<\ell}\left(1 - 2c^\dagger_{\ell'}c_{\ell'}\right)(c_\ell + c^\dagger_\ell)$
- Diagonalize Hamiltonian via Bogoliubov transformation $c^\dagger_q = u_q\gamma^\dagger_q + iv_q\gamma_{-q}$
- $H_I = \sum_q \epsilon_q\left(\gamma^\dagger_q\gamma_q - \frac{1}{2}\right),\ \epsilon_q = 2J\sqrt{1 + h^2 - 2h\cos qa}$
- Continuum limit ($a \to 0, J \to \infty, h \sim 1$): relativistic *free* fermions!
- $\epsilon_q = \sqrt{c^2q^2 + \Delta^2}$ where $\Delta = 2J\,|\,h - 1\,|,\ c = 2J$

Quantum Disordered Regime: Semiclassical Analysis

- Semiclassical limit of QD ($T \ll -\Delta$): $\sigma^z(x,t) = (2cZ|\Delta|^{1/4})^{1/2}(\psi(x,t) + \psi^\dagger(x,t) + \ldots)$

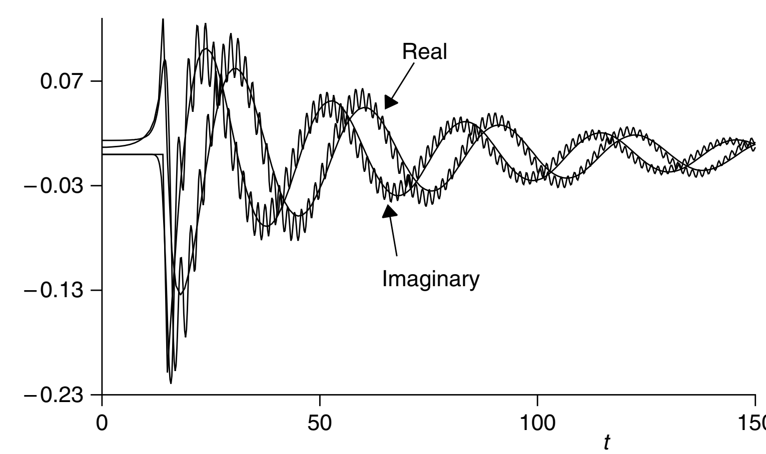
- Find $C(x,t) = \langle \sigma^z(x,t)\sigma^z(0,0)\rangle_{T=0}\Phi_R\left(x/\xi, t/\tau_\varphi\right)$



- Find $\tau_\varphi \sim e^{|\Delta|/T}/T$, Φ_R scaling form (same for RC)

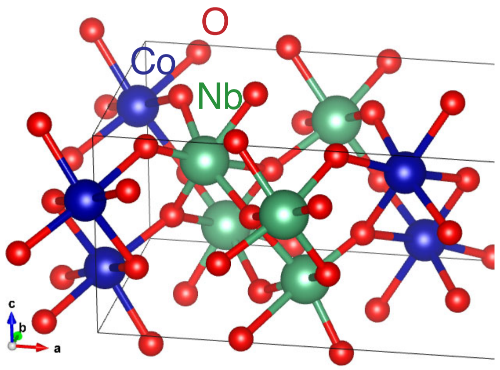
- Fourier transform $S(q,\omega) \approx \dfrac{2cZ|\Delta|^{1/4}}{\epsilon_q}\dfrac{1/\tau_\varphi}{(\omega-\epsilon_q)^2 + (1/\tau_\varphi)^2}$

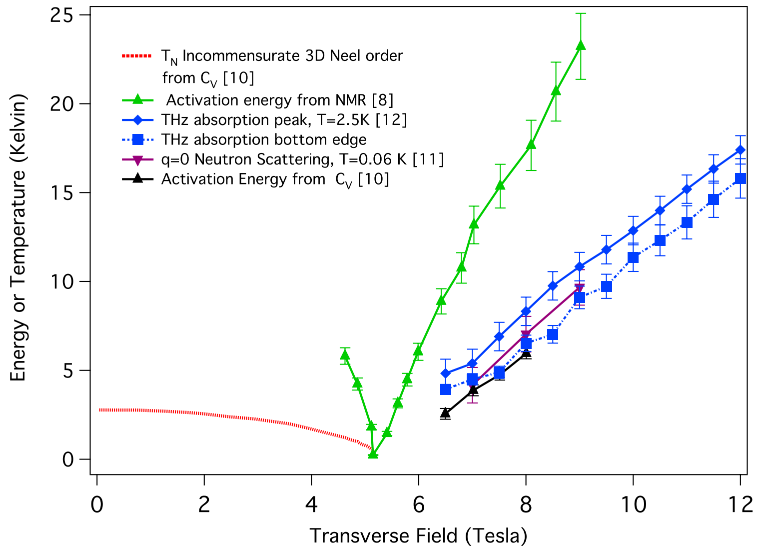
- $S(q,\omega) \approx \dfrac{2T}{\omega}\Im\chi(q,\omega),\ \omega \ll T$



Experiments

- $CoNb_2O_6$ zigzag chain of Co^{2+} ions with effective s=1/2 moment
- Dominant NN interaction along c axis almost ideal TFIC for intermediate T
- In paramagnetic regime determine Δ via THz absorption, neutron scattering, NMR
- NMR relaxation rate defined as $\frac{1}{T_1} = \lim_{\omega \to 0} \frac{2T}{\omega} \int \frac{dq}{2\pi} |a_{hf}|^2 \Im \chi(q, \omega)$
- Differs with other measurements by factor of 2!





A. W. Kinross et. al. Phys Rev X **4**, 031008 (2014)

What went wrong?

- Is semiclassical continuum $\mathfrak{I}\chi(\omega)$ consistent with lattice model?
- $\mathfrak{I}\chi(\omega) = \frac{1}{Z}\sum_{n,m=0}^{\infty}\frac{1}{n!m!}\sum_{\{q\}\{p\}} |\langle q_1,\ldots,q_n|\sigma_0^z|p_1,\ldots,p_n\rangle|^2\,\delta(\omega - E(p) - E(q))(e^{-E(p)/T} - e^{-E(q)/T})$
- $\langle q_1,\ldots,q_n|\sigma_0^z|p_1,\ldots,p_n\rangle$ "form factor": σ^z can take $n \rightarrow m$ fermions, $n - m =$ odd
- From lattice model single fermion energies bounded: $\epsilon_q \in (\epsilon_{min}, \epsilon_{max}) \equiv 2J(h-1, h+1)$
- $\omega \rightarrow 0$, $n \rightarrow m$ process must have $E_i = E_f \in (m\Delta, \epsilon_{max})$

Universal Scaling Result

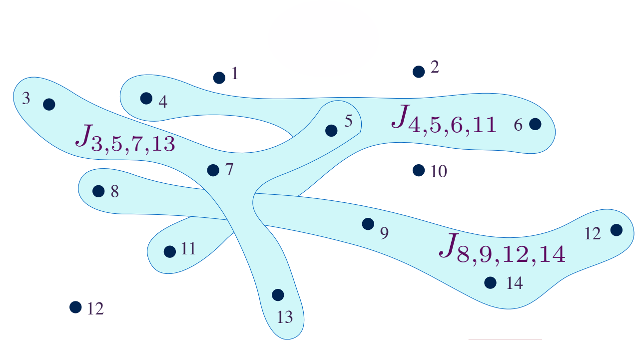
- Allowed processes determined by $h < (m+n)(m-n)$
- For $h < 3$ dominated by $1 \to 2$ processes
- Evaluate integrals for low momentum limit of dispersion, in continuum limit $T \ll \Delta \ll J$

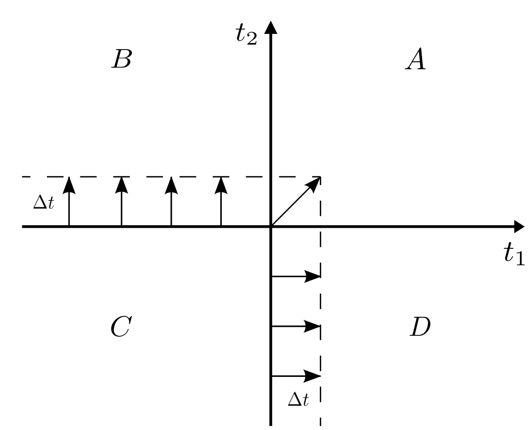
$$\frac{1}{T_1} = |a_{hf}|^2 \frac{Z}{T^{3/4}} \Phi_2(T/\Delta),\ \Phi_2(T/\Delta) = \frac{3\sqrt{3}}{\sqrt{2\pi}} (T/\Delta)^{11/4} e^{-2\Delta/T}$$

- Universal result experimentally testable prediction for scaling and pre-factor!

Non-equilibrium Dynamics in the Sachdev-Ye-Kitaev Model

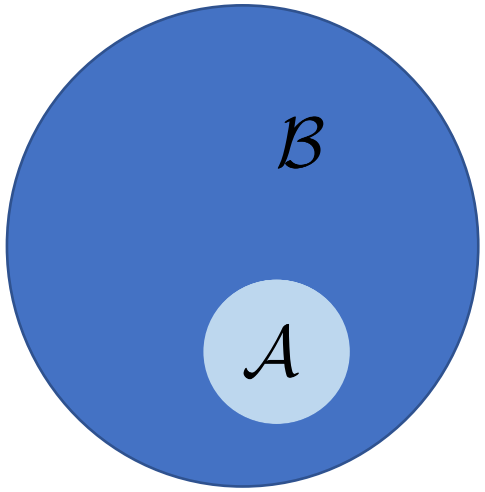
Based on A. Eberlein, V. Kasper, S. Sachdev, and J.S. "*Quantum quench of the Sachdev-Ye-Kitaev model*", Phys. Rev. B. **96**, 205123 (2017), arXiv:1706.07803.





Thermalization in Closed Quantum Systems

- Thermalization/Ergodicity: equivalence of time and ensemble averages
- System described in terms of conserved quantities and initial conditions lost
- Correlation functions obey FDT/KMS conditions
- Apparent paradox: unitary evolution does not lose information
- Resolution: quantum system acts as own heat bath for subsystems
- Information locally inaccessible but preserved globally



What is a Quantum Quench?

- Quantum quench:
  - System starts in pure or thermal state evolving unitarily under one Hamiltonian
  - Hamiltonian changes suddenly (short time compared to energy gap) adding energy to system
  - System continues evolving unitarily under new Hamiltonian, long times expect to end in steady state
- Nature of final state depend on conserved quantities, nature of interactions
- If thermal T determined by conservation of energy post quench

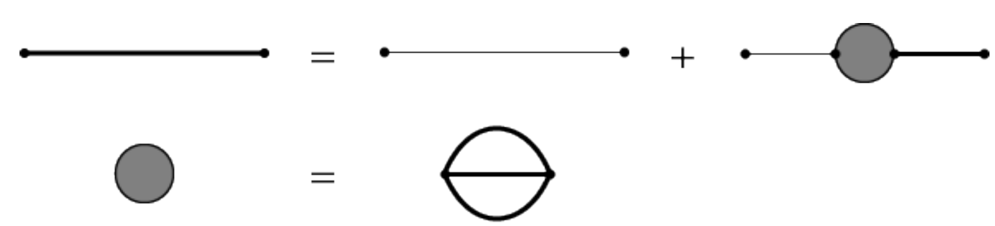
The SYK Model

- Generalized SYK model: N flavors of Majorana fermions with q-body interactions in 0+1 dim

- $$H = (i)^{\frac{q}{2}} \sum_{1 \leq i_1 < i_2 < \ldots < i_q \leq N} j_{i_1 i_2 \ldots i_q} \psi_{i_1} \psi_{i_2} \ldots \psi_{i_q},\ j_{i_1 i_2 \ldots i_q} \sim \mathcal{N}\left(0, \frac{J^2 (q-1)!}{N^{q-1}}\right),\ \{\psi_i, \psi_j\} = 0$$

- Solvable in large N limit $G(\tau_1, \tau_2) \equiv \langle T_\tau \psi(\tau_1) \psi(\tau_2) \rangle$ from Schwinger Dyson eqns

$$G^{-1}(\omega) = -i\omega - \Sigma(\omega),\ \Sigma(\tau_1, \tau_2) = J^2 [G(\tau_1, \tau_2)]^{q-1}$$



- Strong coupling $\beta J \gg 1$ SD eqns invariant under reparameterizations: $\tau \to f(\tau)$

Anomalous Low Energy Properties

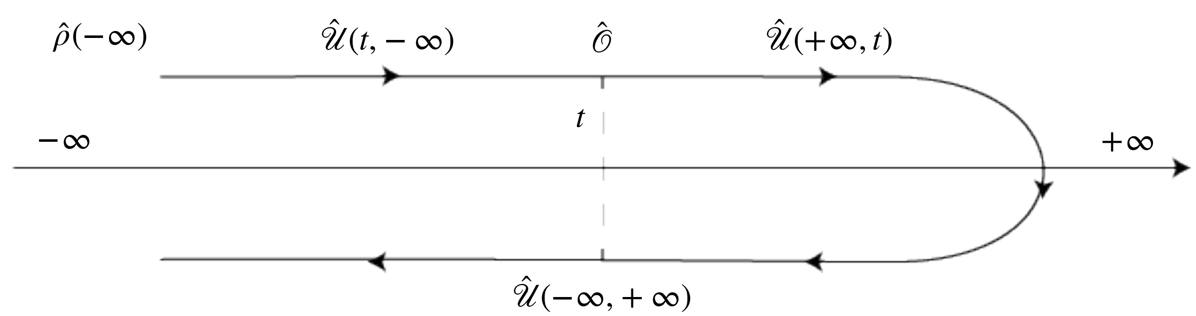
- For $q = 2$, $A(\omega)$ given by Wigner's semicircle Fermi's Golden rule $\tau_{qp} \sim T^{-2}$
- "Conformal" Green's function with spectral density $A(\omega) \sim \omega^{\frac{2}{q}-1}$
- Show from Fermi's Golden rule for $q = 4$, $\tau_{qp} \sim T^{-1}$
- SYK a quantum critical state without quasiparticles and extensive ground state S
- Reparameterization related to local criticality ($z \to \infty$)
- Spontaneously *and* explicit breaking of symmetry leads to Schwarzian effective action and connections to nearly AdS_2 gravity!

Quench Protocol

- How do quantum critical states without quasiparticles behave out of equilibrium?
- SYK a good model to test
- Probe by quantum quench: sudden change to Hamiltonian adds energy to system
- Consider $H(t) = i\sum_{i<j} j_{2,ij} f_2(t)\psi_i\psi_j - \sum_{i<j<k<l} j_{4,ijkl} f_4(t)\psi_i\psi_j\psi_k\psi_l$
- Quench from system with quasiparticles to state without quasiparticles

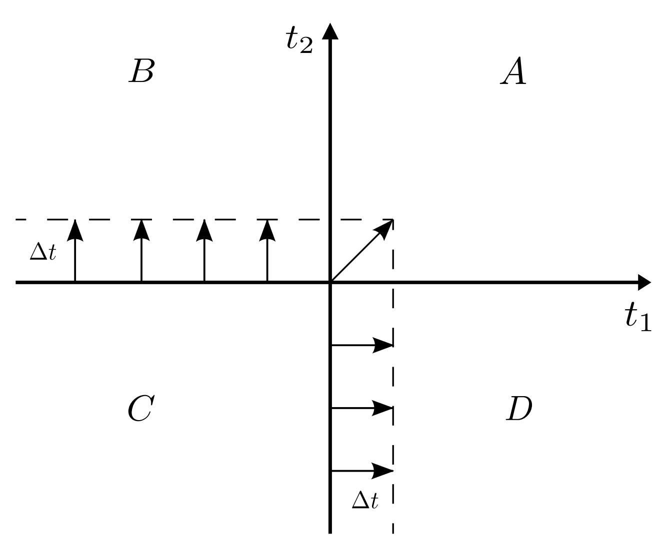
Closed time Contour

- Adiabatic thm: if perturbation turned on slowly, final and initial state differ by phase
- Not true out of equilibrium, can keep track of full time evolution with CTC
- Define fields $\psi^{\pm}$ on forward (backward) legs
- Correlation functions: $iG^{>}(t_1, t_2) = \langle \psi^{-}(t_1)\psi^{+}(t_2)\rangle$, $iG^{>}(t_1, t_2) = \langle \psi^{-}(t_1)\psi^{+}(t_2)\rangle$
- Independent in general but $G^{>}(t_1, t_2) = -G^{<}(t_2, t_1)$ *always* for Majorana fermions



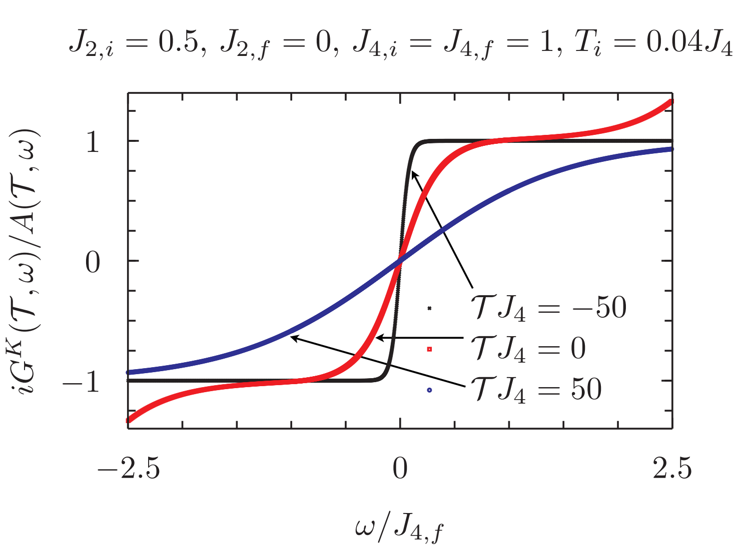
Out of Equilibrium Evolution

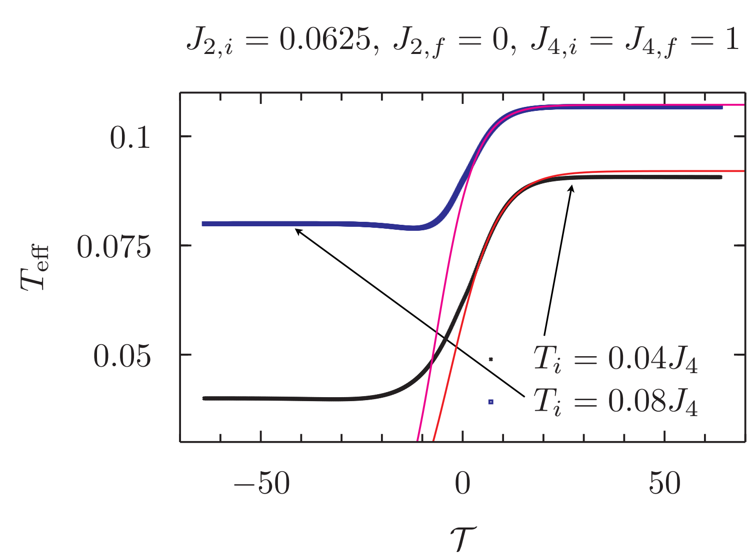
- Use $G^{>}$ and $G^{<}$ to construct all other Green's functions
- $G^R(t_1, t_2) = \Theta(t_1 - t_2)(G^{>} - G^{<}),\ G^A(t_1, t_2) = \Theta(t_2 - t_1)(G^{<} - G^{>}),\ G^K = G^{>} + G^{<}$
- Kadanoff-Baym equations for $G^{>(<)}$ along contour
- $G^{>} \otimes G_0^{-1} = \left(G^R \otimes \Sigma^{>} + G^{>} \otimes \Sigma^A\right),\ G_0^{-1} \otimes G^{>} = \left(\Sigma^R \otimes G^{>} + \Sigma^{>} \otimes G^A\right)$
- Quadrants B, D propagate information from C (pre-quench) to A (post-quench)

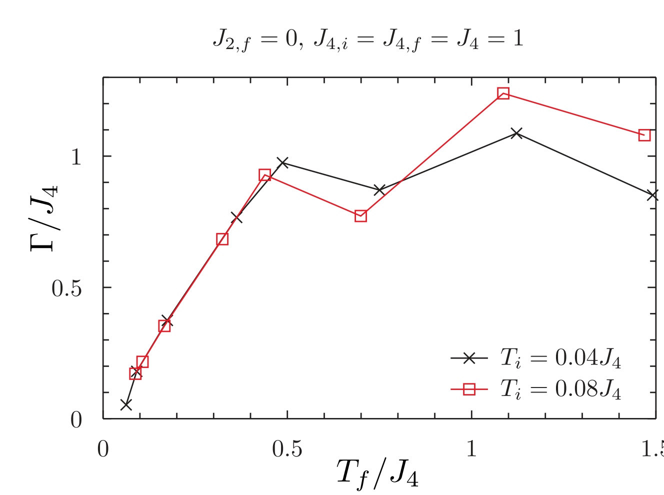


Numerical Study

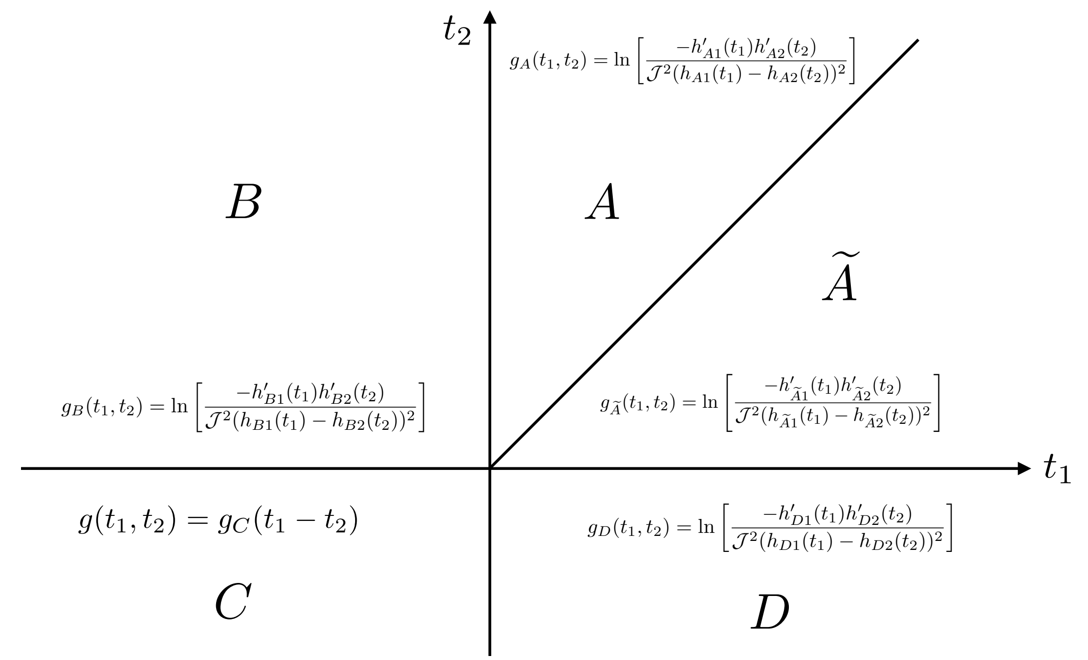
- Absolute time: $\mathcal{T} = t_1 + t_2$ relative time $t = (t_1 - t_2)/2$,
- Near equilibrium Wigner transform $f(\mathcal{T}, t) \to f(\mathcal{T}, \omega)$ to look at small ω's
- Near equilibrium vary slowly with $\mathcal{T}$ and FDT implies: $\dfrac{iG^K}{A}(\mathcal{T}, \omega) \approx \tanh(\beta_{eff}(\mathcal{T})\omega/2)$
- Fit $\beta_{eff}(\mathcal{T}) = \beta_f + \alpha \exp\left(-\Gamma \mathcal{T}\right)$ and find $\Gamma \sim T$ for low T







Large q Limit



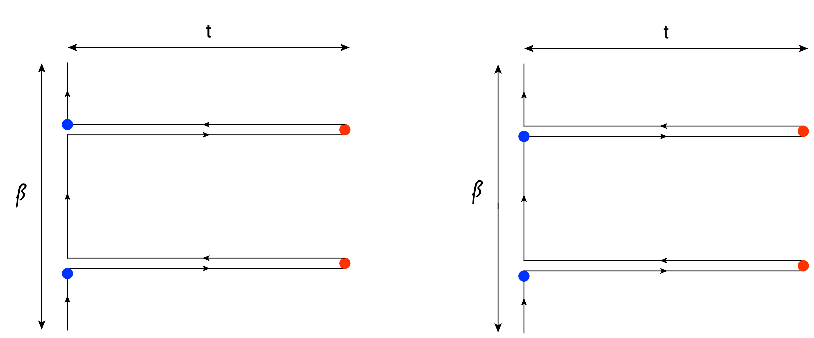
- Consider quench from $q + pq \to pq$ SYK for large q
- Kadanoff-Baym equations exactly solvable for $p = 2$
- Impose pre-quench solution in C
- $g(t_2, t_1) = [g(t_2, t_1)]^*$ everywhere from causality
- $SL(2,C)$ invariance: $h(t) \to \dfrac{ah(t) + b}{ch(t) + d}$ $ad - bc = 1$, and $g(t, t) = 0$
- Use to fix BCs and find $h_{A1}(t), h_{A2}(t)$: final solution which *only* depends on t *not* $\mathscr{T}$!

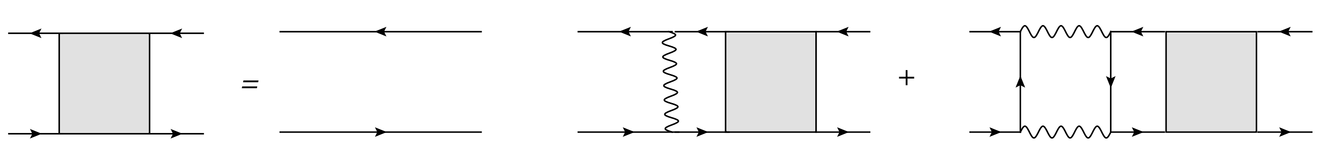
Remarks and Connection to Gravity

- In general conjecture that $\Gamma \sim \mathscr{C}(q)T$ where $\mathscr{C}(\infty) = 0$
- Instantaneous thermalization of $G^{>}$ in large q limit!
- $h_{A1}(t), h_{A2}(t)$ solutions to EOM of Schwarzian action!
- Local thermalization connected to reparameterization modes
- *Any* theory with Schwarzian effective action should *locally* thermalize instantaneously
- However quantities with higher order correlations can take long time to thermalize!

Thermalization and chaos in a conformal gauge theory

Based on J. S. and B. Swingle. “Thermalization and chaos in QED$_3$”, Phys. Rev. D 99, 076007 (2019), arXiv: 1901.04984.





Thermalization and chaos

- Thermalization a multi-step process in strongly interacting quantum systems
  - Local relaxation (early)→Global scrambling of information (intermediate)→Build up of complexity (late)
- Scrambling shown in exponential decay of OTOC:

$$\frac{\langle W(t,\mathbf{x})V(0)W(t,\mathbf{x})V(0)\rangle}{\langle W(t,\mathbf{x})W(t,\mathbf{x})\rangle\langle V(0)V(0)\rangle} \sim 1 - e^{\lambda_L(t-\mathbf{x}/v_B)}$$

- Rigorous upper bound $\lambda_L \leq \frac{2\pi}{\beta}$, v_B maximum speed of information propagation
- Expect quantum critical systems to be fast scramblers i.e. $\lambda_L \sim \mathscr{C}\frac{2\pi}{\beta}$

QED in 2+1 dimensions

- N_f massless fermions coupled to U(1) gauge field:

$$\mathscr{L} = -\frac{1}{4}F^2 + \sum_{a=1}^{N_f} i\bar{\psi}_a\gamma^\mu(\partial_\mu - ieA_\mu)\psi_a$$

- QED_3 emerges from lattice modes where order parameters fractionalize

- Coupling e is *relevant*, flow to strongly interacting IR fixed point

- Limit of large N_f, fluctuations in F suppressed expansion about *free* theory in $1/N_f$

- Expansion breaks down for some unknown N_c

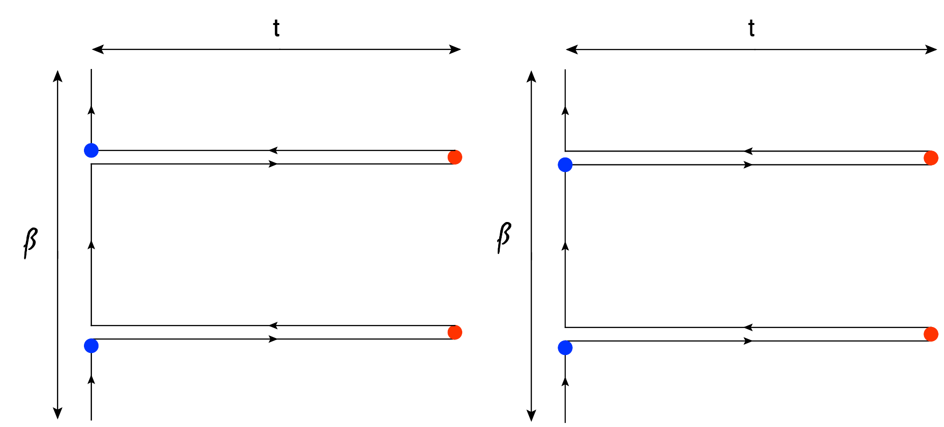
Extracting Chaos Exponents

- OTOC contained in squared anti-commutator,

- $$\mathscr{F}(t,\mathbf{x}) = \frac{1}{N_f^2}\sum_{a,b=1}^{N_f} \mathrm{Tr}\left[\sqrt{\hat{\rho}}\{\psi_a(t,\mathbf{x}),\bar{\psi}_b(0)\}\sqrt{\hat{\rho}}\{\psi_a(t,\mathbf{x}),\bar{\psi}_b(0)\}^\dagger\right]$$

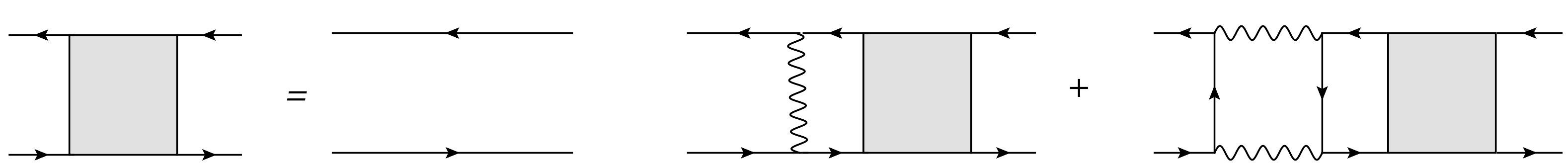
- Grows ballistically $\sim e^{\lambda_L(t-\mathbf{x}/v_B)}$

- Time ordering on double contour

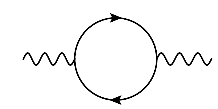


- Vertical lines: retarded propagators, horizontal lines: Wightman function

- Represented as ladder diagram with two $O(1/N_f)$ rungs

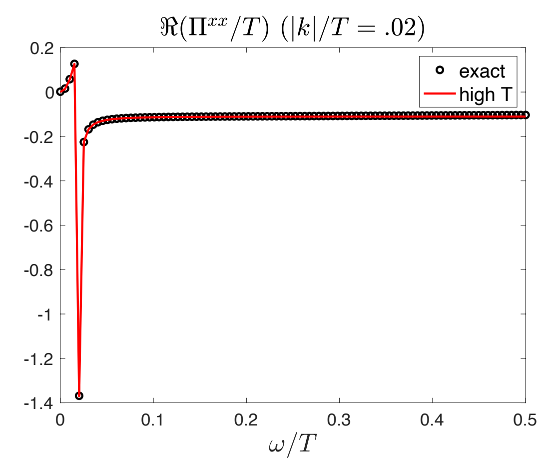


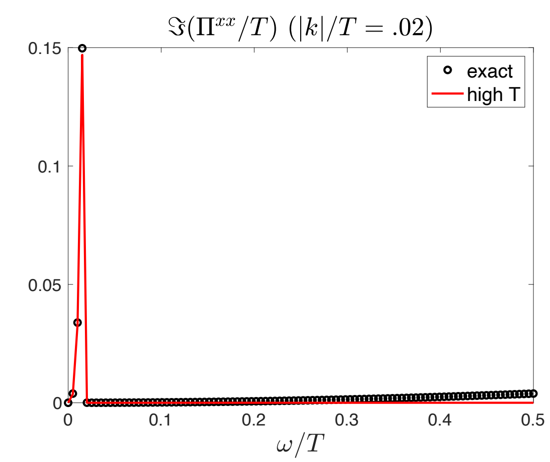
Propagators

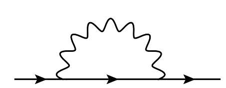
- Photon propagator: nonperturbative $O(1)$ object 

- Longitudinal and transverse part, form restricted by current conservation

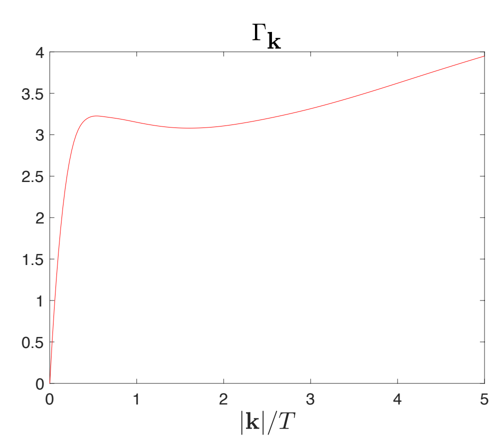
- Develops thermal $m_{th}^2 = \dfrac{e^2 T \log 2}{\pi}$





- Fermion mass $O(1/N_f)$ 

- Imaginary part $\Gamma_{\mathbf{k}} \sim \dfrac{T}{N_f}$ gives finite lifetime and suppresses chaos



Bethe-Salpeter Equation

- Long times, legs on shell, exponential growth
- Ladder→two body wave-function, eigenstate of "projection" operator
- Do projection and get eigenvalue equation for two part function

$$(-i\nu + is\delta\epsilon_{\mathbf{q}} + 2\Gamma_{\mathbf{k}})\psi_s(\nu, \mathbf{q}, \mathbf{k}) = \sum_{s'=\pm} \frac{1}{N_f} \int_{\mathbf{k}'} K_{ss'}(\mathbf{k}, \mathbf{k}')\psi_{s'}(\nu, \mathbf{q}, \mathbf{k}')$$

- Largest eigenvalue controls chaos: $\lambda_L = \mathscr{C}\dfrac{2\pi T}{N_f}, \mathscr{C} \approx 0.65$
- Never violates bound despite breakdown of large N_f expansion

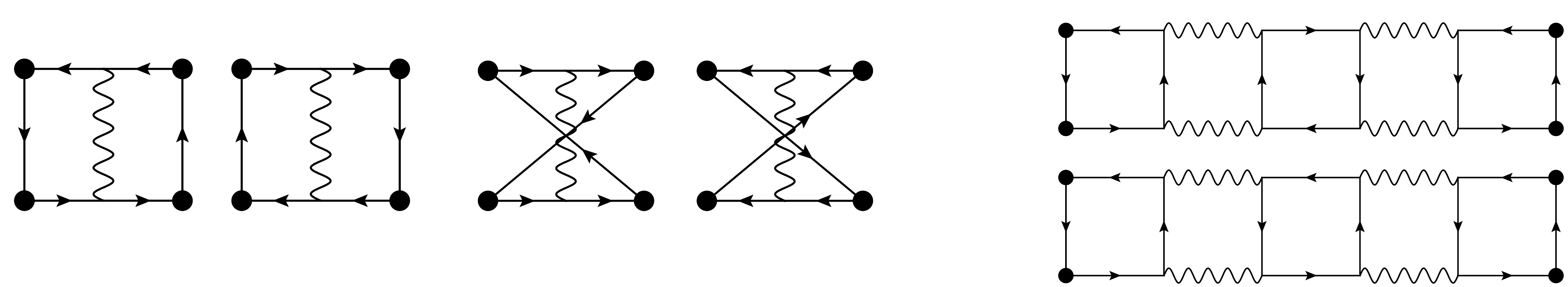
Chaos and Gauge Invariance, Symmetry Breaking

- Could consider gauge invariant quantity
$$\mathscr{C}(t,\mathbf{x}) = \delta_{\mu\rho}\delta_{\nu\sigma}\frac{1}{N_f^2}\sum_{a,b=1}^{N_f} \mathrm{Tr}\left[\sqrt{\hat{\rho}}[j_a^\mu(t,\mathbf{x}), j_b^\nu(0)]\sqrt{\hat{\rho}}[j_a^\rho(t,\mathbf{x}), j_b^\sigma(0)]^\dagger\right]$$

- A^μ and j^μ correlators related by current conservation

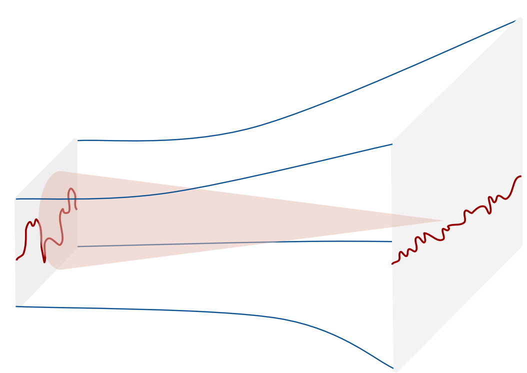
- All diagrams in $\mathscr{F}(t,\mathbf{x})$ contained in $\mathscr{C}(t,\mathbf{x})$ + additional suppressed by $1/N_f$, chaos should grow at least as fast as for $\mathscr{F}(t,\mathbf{x})$

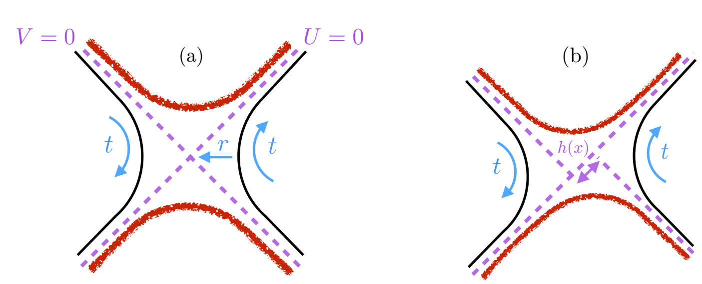
- If dynamical mass generated through chiral symmetry breaking expect $\lambda_L \sim e^{-m/T}$



Bounding Transport in Disordered Holographic Matter

Based on Andrew Lucas and J. S. "*Charge diffusion and the butterfly effect in striped holographic matter*", JHEP 1610, **143** (2016), arXiv:1608.03286.





Bounding Transport in Matter Without Quasiparticles

- Widely believed that anomalous transport in strange metal phase of cuprates related to presence of QCP at $T = 0$

- Points to saturation of some quantum bound

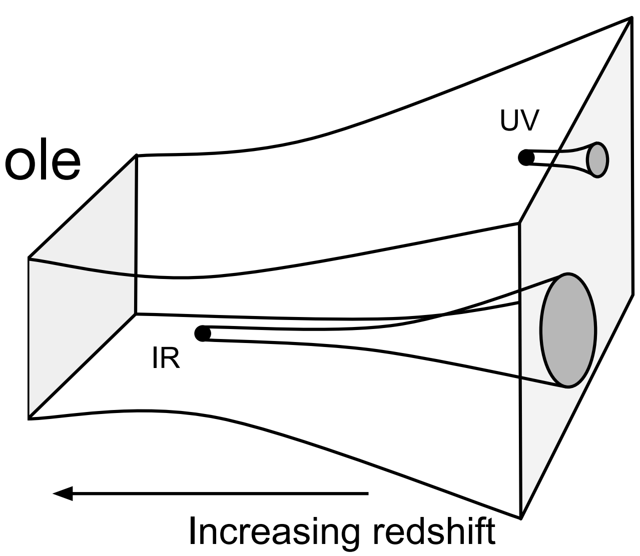
- How to precisely formulate bound and relate to transport?

- Conjectured that $D \geq \mathscr{C}\dfrac{v^2}{T}$ for charge diffusion, v unspecified

- In certain class of holographic IR fixed points, $D = \mathscr{C}\dfrac{v_B^2}{T}$,

Holographic Quantum Matter

- AdS/CMT: in some limit strongly coupled QFT may be described as boundary theory of *classical* theory of gravity
- Radial dimension→energy scale
- Black hole solutions to Einstein equations where $T_{QFT} = T_H$
- RG flow from UV boundary fixed point→IR horizon fixed point
- Universal low energy effective theory on horizon of black hole



S. Hartnoll, A. Lucas, and S. Sachdev, *Holographic Quantum Matter.*

What do Black Holes have in Common with Quantum Matter?

- Classical black holes are thermodynamic
  - Surface gravity κ constant (0th law)
  - $\delta M = \dfrac{\kappa}{8\pi G_N}\delta A$ (1st law)
  - $\delta A \geq 0$ (2nd law)
- Black holes are dissipative, fast relaxation $\tau \sim \dfrac{\hbar}{k_B T}$, Hawking radiation
- Exotic quantum matter without quasiparticles may share properties of black holes!

Constructing “Bottom” up Models

- To study universal properties construct gravity dual via Landau-Ginzburg like procedure

- Operator in QFT→bulk field, global symmetry→gauged symmetry

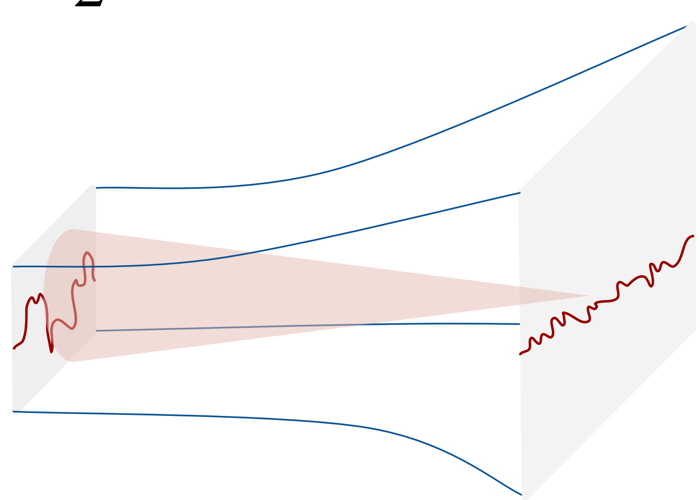
- EMD: $S = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 2(\partial\Phi)^2 - \frac{V(\Phi)}{L^2}\right) - \frac{Z(\Phi)}{4e^2}F^2\right]$

- F Maxwell field (conserved charge), Φ dilaton (turns gauge coupling dynamical)

- Emergent IR geometry describes zoo of exotic quantum matter with various z and hyperscaling violating exponent θ

Adding Disorder to Holographic Matter

- Transport bounds should be robust in presence of disorder
- Hard to compute in disordered QFTs but relatively simple for holographic matter
- Add disorder through striped dilaton: $\Phi(x, r \to 0) = H(x) r^{d+1-\Delta} L^{-d/2} + \ldots$
- For relevant dilaton i.e. $\Delta < d + 1$ disorder distorts IR geometry
- Solve Einstein equation $R_{MN} - \dfrac{R}{2} g_{MN} - T_{MN} = 0$ for new black hole geometry



Fluid Gravity Expansion

- For $\partial_x H/H \sim \xi^{-1}$ and $1 \ll \xi T$ "hydrodynamic" disorder

- Fluid gravity expansion in coordinates regular at horizon, new metric

$$ds^2 = -2\tilde{a}(r; H(x))dvdr - a(r; H(x))dv^2 + b(r; H(x))d\mathbf{x}^2 + O(\xi^{-1})$$

$$\Phi = \phi(r; H(x)) + O(\xi^{-1})$$

- Can also use Kruskal coordinates which cover whole spacetime

$$ds^2 = 2A(UV; H(x))dUdV + B(UV; H(x))d\mathbf{x}^2 + O(\xi^{-2})$$

Conductivity

- Charge diffusion coefficient $D = \sigma/\chi$
- σ from linear response: add electric field perturbation $A = -Evdx + \tilde{A}$
- Maxwell equation for gauge field $\partial_M\left(\sqrt{-g}Z(\Phi)F^{MN}\right) = 0$
- Leads to current $J_x = e^{-2}\sqrt{-g}ZF^{xr}$, radially conserved $J_x \leftrightarrow \langle J_x \rangle$ QFT
- Ohm's law $\sigma = \langle J_x \rangle / E$
- In terms of the local conductivity $\sigma = 1/\mathbb{E}[1/\tilde{\sigma}(x)]$,

Susceptibility

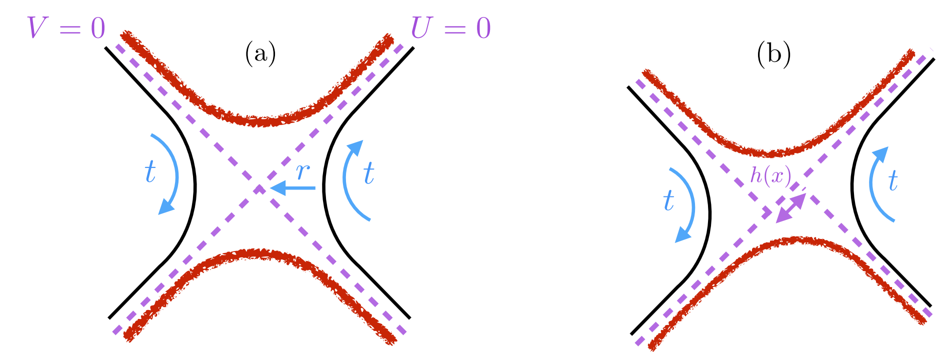
- $\chi \equiv \dfrac{\partial \mathbb{E}[n]}{\partial \mu}$ where $n = \langle J_t \rangle$
- Find $n \approx -\dfrac{1}{e^2} Z F^{vr}$ in fluid gravity limit
- Impose $A(r=0, x) = \mu dv$, evaluation of χ involves integral through full geometry
- Find dominant contribution from horizon geometry if scaling dimension of χ is relevant
- In the end find $D \sim \mathscr{C} \dfrac{1}{4\pi T \mathbb{E}[H^{\eta}]\mathbb{E}[H^{-\eta}]}$ η scaling exponent from IR geometry

Butterfly velocity

- Solve Einstein equation with shockwave term added to stress tensor

- Use WKB like approximation and find

$$h \sim e^{\lambda_L (t - |x|/v_B)}$$



- $1/v_B = \mathbb{E}[1/\tilde{v}_B]$ and to $O(\xi^{-1})$ $\tilde{v}_B$ is constant

- Cauchy-Schwarz inequality implies $\mathbb{E}[H^\eta]\mathbb{E}[H^{-\eta}] \leq 1$ implying $D \leq \mathscr{C}\dfrac{v_B^2}{T}$

- Conclude v_B not a good velocity to bound D

Outlook

- Dynamics of QC systems driven by interplay of quantum and thermal fluctuations
- Lack of separation of timescales makes it hard to study QC systems in controlled setting
- When system is dual to a free theory, get nice results which can be related to experiments
- Otherwise can use analytical tools like large N expansion, Keldysh, AdS/CMT
- Large N limit may be issue, gravity dual seems removed from microscopic condensed matter systems
- However, approaches provide promising qualitative insights for future directions

Other work: Computation as Emergent Phenomena in Neural Networks

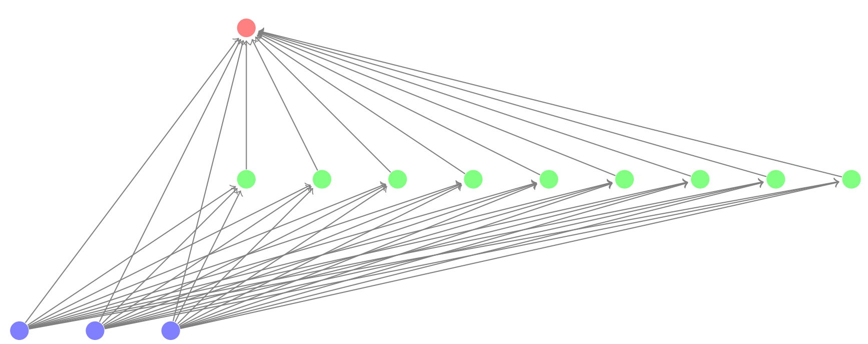
J. S., M. Advani, and H. Sompolinsky "*A New Role for Sparse Expansion in Neural Networks*"
J. S. and H. Sompolinsky "*Associative Memory of Structured Knowledge*" to appear

Neural Networks are Interacting Many-Body Systems

- Neural network: large network of two state “neurons" which transition when input reaches firing threshold
- “Synapses”: connections between neurons
- Neural network coarse grained description of biological neurons and synapses
- Provide insight on how principles of computation emerge from properties of network
- Useful analogies with statistical physics of disordered systems to study equilibrium and dynamical properties

A New Role For Sparse Expansion in Neural Networks

- Sensory processing in brain: sequence of representations
- Sparse expansion: low dimensional dense activity→high dimensional sparse activity
- Neurogenesis: process of creating and recycling neurons in brain
- Can sparse expansion improve learning if synapse later pruned?
- Find in simple model, expanding network gives it capacity to find better solutions
- Related to slack regularization



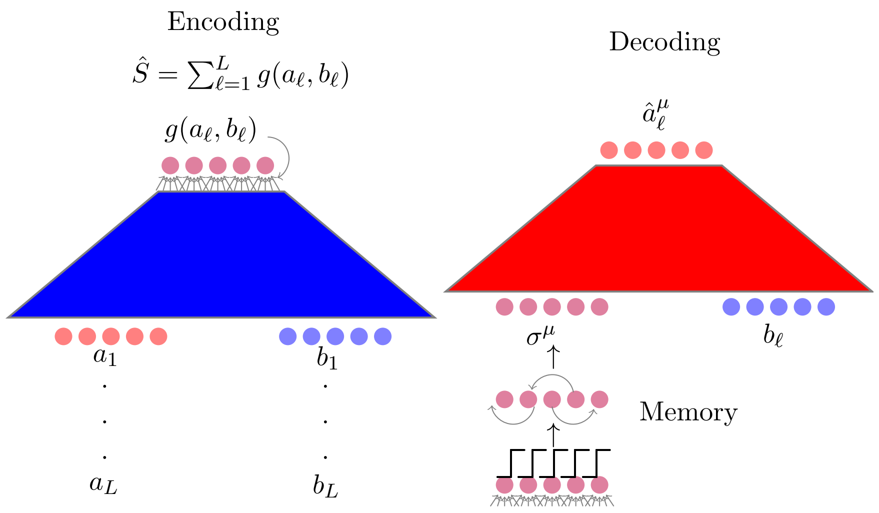
Associative Memory of Structured Knowledge

- Model to store structured knowledge (memories of multiple associated items) in associative memory

- Memories stored in synapses but binding relations between items in neurons

- Full structure stored as *single* attractor instead of series of attractors

- Number of stored structures, scales linearly with network size. Length of structures affects the quality of retrieval. Novel scaling law: $L/N^{\frac{1}{3}} \sim 1$



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